

# Precision radiative corrections to the Dalitz plot of baryon semileptonic decays including the spin-momentum correlation of the decaying baryon and the emitted charged lepton

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## Abstract

We calculate the radiative corrections to the angular correlation between the polarization of the decaying baryon and the direction of the emitted charged lepton in the semileptonic decays of spin one-half baryons to order  $(\alpha/\pi)(q/M_1)$ . The final results are presented, first, with the triple integration of the bremsstrahlung photon ready to be performed numerically and, second, in an analytical form. The results are useful in the analysis of the Dalitz plot of precision experiments involving light and heavy quarks and is not compromised to fixing the form factors at predetermined values. It is assumed that the real photons are kinematically discriminated. Otherwise, our results have a general model-independent applicability.

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In this paper we want to obtain precision radiative corrections (RC) to the  $\hat{s}_1 \cdot \hat{l}$  correlation of baryon semileptonic decays (BSD),  $A \rightarrow B l \bar{\nu}_l$  (where  $A$  and  $B$  are spin one-half baryons,  $l$  is the charged lepton, and  $\nu_l$  is the accompanying antineutrino or neutrino as the case may be,  $\hat{s}_1$  is the polarization of  $A$  and  $\hat{l}$  is the momentum direction of  $l$ ) to order  $(\alpha/\pi)(q/M_1)$ , where  $q$  is the momentum transfer and  $M_1$  is the mass of  $A$ . However, it is not possible to obtain the results for this correlation by a simple change of variables in the final results for other observables. One is forced to return to the early steps of the analysis and perform there the appropriate change of variables. A program must be followed [1]. To this order of approximation, the model independence of RC is ensured following the approach of Refs. [2, 3]. We will assume that even if real photons go undetected, they can be kinematically discriminated so that we will limit our calculations to the three-body region of BSD [1].

The calculation we envisage here is very long and tedious. However, a substantial amount of effort can be saved by keeping a close parallelism with previous calculations. In a recent work [4] we obtained the RC to the above order of precision for the  $\hat{s}_1 \cdot \hat{p}_2$  correlation (where  $\hat{p}_2$  is the direction of  $B$ ). The analysis of this Ref. [4] can be applied here step by step and if we use the same conventions and notation the final results for the  $\hat{s}_1 \cdot \hat{l}$  correlation are in exact parallelism with the final results for the  $\hat{s}_1 \cdot \hat{p}_2$  correlation of this reference.

This allows us to present directly the compact form of the final results of the RC to the Dalitz plot with the  $\hat{s}_1 \cdot \hat{l}$  correlation explicitly exhibited, namely,

$$d\Gamma_i = d\Omega \left[ A'_0 + \frac{\alpha}{\pi} \Theta_{iI} - \hat{s}_1 \cdot \hat{l} \left( A''_0 + \frac{\alpha}{\pi} \Theta_{iII} \right) \right], \quad (1)$$

where the index  $i = C, N$  corresponds to charged decaying baryon (CDB) and the neutral decaying baryon (NDB) cases, respectively. Here  $\Theta_{iI} = B'_1(\Phi_i + I_{i0}) + B'_{i1}\Phi'_i + C'_i$  and  $\Theta_{iII} = B'_2(\Phi_i + I_{i0}) + B'_{i2}\Phi'_i + C^{(s)}_i$ .

In Eq. (1) both the virtual and the bremsstrahlung RC are included. The first two terms within the curly brackets correspond to the unpolarized Dalitz plot. We shall only display the new results here. The explicit form of all other terms can be readily identified using Ref. [4]. Thus,  $B'_2 = Ep_2y_0\tilde{Q}_6 + El\tilde{Q}_7$ ,  $B''_{C2} = Ep_2y_0Q_8 + ElQ_9$ , and  $B''_{N2} = M_1p_2y_0Q_{N8} + M_1lQ_{N9}$ . The other new contribution  $C^{(s)}_C$  of the polarized terms in  $\Theta_{CII}$  consists of the sum of three terms, namely,

$C_C^{(s)} = p_2 l / (2\pi) \sum_{R=I}^{III} \int_{-1}^{y_0} dy \int_{-1}^1 dx \int_0^{2\pi} d\varphi_k |M_R|^2 / D$ , where

$$|M_I|^2 = \frac{\beta^2(1-x^2)}{2(1-\beta x)^2} \frac{E}{M_1} \left\{ -\frac{M_1 D}{l} \tilde{Q}_6 + M_1 x Q_9 + \frac{p_2 y}{E} [-D + E(1 + \beta x)] Q_{10} \right. \\ + [(p_2 y + l)(1 - \beta x) + \beta(E + E_\nu^0 + p_2 x y - D)] Q_{11} \\ + \left[ \frac{p_2 y + l}{E} D - l(1 - \beta x) + \beta(E + E_\nu^0 - p_2 x y) \right] Q_{12} + l Q_{13} \\ \left. + \left[ -\frac{p_2 y + l}{E} D - l(1 - \beta x) + \beta(E + E_\nu^0 + p_2 x y) \right] Q_{14} \right\},$$

$$|M_{II}|^2 = \frac{1}{2(1-\beta x)} \left\{ p_2 y \tilde{Q}_6 + l \tilde{Q}_7 + R_1 p_2 y Q_8 + \{R_1 l + [R_2 E_\nu + w(R_1 + 1)]x\} Q_9 \right. \\ - \frac{p_2 y}{M_1} \left[ \frac{D\omega}{E} + R_2(\mathbf{p}_2 \cdot \hat{\mathbf{k}} + lx) - 2(1 + R_1)\omega \right] Q_{10} + \frac{l}{M_1} w Q_{13} - R_2 E_\nu x Q_{15} \\ + \frac{p_2 l}{M_1} \left\{ \frac{2\omega}{p_2} (1 - \beta x) - \hat{\mathbf{p}}_2 \cdot \hat{\mathbf{k}} \left[ R_2 + \frac{\omega}{E} \right] - R_2 \frac{x}{l} [\hat{\mathbf{p}}_2 \cdot (\mathbf{k} - \mathbf{p}_\nu) - ly] + \frac{\omega}{E} xy \right\} Q_{11} \\ + \frac{p_2}{M_1} \left\{ R_2 \hat{\mathbf{p}}_2 \cdot [\hat{\mathbf{k}}(p_2 y + l + \omega x) - \mathbf{p}_\nu x] + \beta \omega \left[ \frac{D(p_2 y + 2l)}{p_2 l} - \hat{\mathbf{p}}_2 \cdot \hat{\mathbf{k}} - xy \right] \right\} Q_{12} \\ \left. + \frac{p_2}{M_1} \left[ R_2 [-\mathbf{p}_2 \cdot \hat{\mathbf{k}} y + x(p_2 + ly)] - \hat{\mathbf{p}}_2 \cdot \hat{\mathbf{k}} (R_2 l + \beta \omega) + \frac{\omega y}{E} (lx - D) \right] Q_{14} \right\},$$

and

$$|M_{III}|^2 = \frac{1}{1-\beta x} \frac{l}{M_1} \left\{ 2E_\nu(x^2 - 1)Q_{16} + (E_\nu - \beta \hat{\mathbf{l}} \cdot \mathbf{p}_\nu)(1 - x^2)Q_{17} \right. \\ + \beta(p_2 y + l + \omega x)(1 - x^2)Q_{18} + (-E_\nu + D + \hat{\mathbf{l}} \cdot \mathbf{p}_\nu x)Q_{19} \\ + (E_\nu - D - \hat{\mathbf{l}} \cdot \mathbf{p}_\nu x)Q_{20} + \beta[x(D - E_\nu) + \hat{\mathbf{l}} \cdot \mathbf{p}_\nu]Q_{21} \\ + \frac{\omega x}{2l}(D - E_\nu + \beta \hat{\mathbf{l}} \cdot \mathbf{p}_\nu)Q_{22} + \frac{\omega}{2l}[\beta(D - 2E_\nu) + (E_\nu + \beta \hat{\mathbf{l}} \cdot \mathbf{p}_\nu)x]Q_{23} \\ \left. + \frac{E_\nu \omega}{2l}(1 - \beta x)xQ_{24} - \frac{\omega}{2l}(p_2 y + l + \omega x)(1 - \beta x)Q_{25} \right\}.$$

The NDB case contains  $C_N^{(s)} = C_C^{(s)} + C_{NA}^{(s)}$ , where  $C_{NA}^{(s)} = \tilde{C}_I^{(s)} + \tilde{C}_{II}^{(s)} + \tilde{C}_{III}^{(s)}$ . The explicit forms of these latter are  $\tilde{C}_i^{(s)} = D_3 \rho_i + D_4 \rho'_i$ ,  $i = I, II, III$ , with  $D_3 = 2(-g_1^2 + f_1 g_1)$  and  $D_4 = 2(g_1^2 + f_1 g_1)$ , and

$$\rho_I = \frac{p_2^2 l^2 \beta}{\pi M_1} \int_{-1}^{y_0} dy \int_{-1}^1 dx \int_0^{2\pi} \frac{d\varphi_k}{D(1-\beta x)} (-y + x \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}_2), \quad (2)$$

$$\rho'_I = \frac{p_2^2 l}{\pi M_1} \int_{-1}^{y_0} dy \int_{-1}^1 dx \int_0^{2\pi} \frac{d\varphi_k}{D(1-\beta x)} (-y + x \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}_2) (-D + lx), \quad (3)$$

$$\rho_{II} = \frac{p_2^2 l}{4\pi M_1} \int_{-1}^{y_0} dy \int_{-1}^1 dx \int_0^{2\pi} \frac{d\varphi_k}{D} E_\nu \left[ y + \frac{\beta y - \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}_2}{1 - \beta x} x \right], \quad (4)$$

$$\rho'_{II} = \frac{p_2^2 l}{4\pi M_1} \int_{-1}^{y_0} dy \int_{-1}^1 dx \int_0^{2\pi} \frac{d\varphi_k}{D} \left[ \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}_2 + \frac{\beta y - \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}_2}{1 - \beta x} \right] \mathbf{p}_\nu \cdot \hat{\mathbf{l}}, \quad (5)$$

$$\begin{aligned} \rho_{III} = & \frac{p_2 l}{2\pi M_1} \int_{-1}^{y_0} dy \int_{-1}^1 dx \int_0^{2\pi} \frac{d\varphi_k}{D(1 - \beta x)} \left\{ [l(x - \beta) - 2\omega(1 - \beta x)] E_\nu x \right. \\ & \left. - l [E_\nu(1 - \beta x) - \beta \mathbf{p}_\nu \cdot (x \hat{\mathbf{k}} - \hat{\mathbf{l}})] \right\}, \end{aligned} \quad (6)$$

$$\begin{aligned} \rho'_{III} = & \frac{p_2 l}{2\pi M_1} \int_{-1}^{y_0} dy \int_{-1}^1 dx \int_0^{2\pi} \frac{d\varphi_k}{D(1 - \beta x)} \left\{ -[bl(1 - x^2) + 2\omega(1 - \beta x)] \mathbf{p}_\nu \cdot \hat{\mathbf{l}} \right. \\ & \left. - l(E_\nu - \mathbf{p}_\nu \cdot \hat{\mathbf{k}}) + (-\mathbf{l} \cdot \mathbf{p}_\nu + E_\nu l x) \right\}. \end{aligned} \quad (7)$$

The above expressions contain triple integrals that can be integrated numerically. This is our first final result and covers both the CDB and NDB cases.

These integrals can be performed analytically. A convenient rearrangement is  $C_C^{(s)} = \sum_{i=1}^9 Q_{i+5} \Lambda_i + \sum_{i=6}^{15} Q_i \Lambda_{i+4} + \sum_{i=16}^{25} Q_i \Lambda_{i+4}$ . The form factors appear in the  $Q_i$  functions, which are all collected in Appendix A of Ref. [4]. The triple integrals are contained in the  $\Lambda_i$ . In organizing the first summand of  $C_C^{(s)}$  with one running index  $i$  it was necessary to introduce  $\Lambda_2 = \Lambda_3 = 0$ , because  $Q_7$  and  $Q_8$  do not appear in this equation. Skipping details, we get

$$\begin{aligned} \Lambda_1 &= -E p_2 \theta_0, & \Lambda_4 &= \frac{E^2 p_2}{2} (Y_2 - Y_3), \\ \Lambda_5 &= -\frac{E p_2}{2 M_1} [-2Z_1 + \beta l p_2 Y_1 + Z_2 + \beta p_2 (y_0 - 1) \theta_0], \\ \Lambda_6 &= \frac{E p_2}{2 M_1} [l^2 Y_3 - 2\beta l \theta_0 + \beta l (E + E_\nu^0) Y_2 + Z_1], \\ \Lambda_7 &= \frac{p_2 l}{2 M_1} \left\{ p_2 (y_0 - 1) \theta_0 + l [2\theta_0 - E Y_3 + (E + E_\nu^0) Y_2] + p_2 l Y_1 + \frac{Z_2 - Z_1}{\beta} \right\}, \\ \Lambda_8 &= \frac{E p_2 l^2}{2 M_1} Y_2, \\ \Lambda_9 &= -\frac{p_2 l}{2 M_1} \left\{ p_2 (y_0 - 1) \theta_0 + l (2\theta_0 + E Y_3 - (E + E_\nu^0) Y_2) + \frac{Z_2 - Z_1}{\beta} + p_2 l Y_1 \right\}, \\ \Lambda_{10} &= \frac{p_2}{2} \zeta_{11}, & \Lambda_{11} &= \frac{p_2 l^2}{2} \theta_3, \end{aligned}$$

$$\Lambda_{12} = -\frac{p_2}{2} \left[ \frac{Z_3}{E} + \zeta_{10} \right],$$

$$\Lambda_{13} = \frac{Ep_2}{2} \left\{ (E_\nu^0 + 2E - l\beta)(1 - \beta^2)\theta_2 - [(1 - \beta^2)(E_\nu^0 + 3E) + E]\theta_3 + 2E(1 - \beta^2)\theta_4 \right. \\ \left. - 3l\theta_5 - (1 - \beta^2)\theta_6 + (5E - E_\nu^0 - l\beta)\frac{\theta_7}{2E} + \frac{\theta_9}{2E} + l\beta\theta_{10} - \frac{\beta}{2}\theta_{14} - 3\eta_0 \right\},$$

$$\Lambda_{14} = -\frac{p_2}{2M_1} \left[ \frac{p_2 l}{2}(1 - y_0)(\theta_0 - 2\eta_0) - \frac{E_\nu^0}{E}Z_3 + (1 - \beta^2)\zeta_{22} - \frac{\zeta_{31}}{2E} - 2\zeta_{21} \right],$$

$$\Lambda_{15} = -\frac{p_2}{2M_1} \left\{ \frac{p_2 l}{2}\eta_0(y_0 - 1) + l(1 - \beta^2)\chi_{12} - l\chi_{11} - 2l^2\eta_0 + p_2^2(\eta_0 - \gamma_0 + l\beta\theta_3) \right. \\ \left. - \frac{\zeta_{21}}{2} + \frac{X_3 - X_4}{l} + \frac{\chi_{30}}{2l} \right\},$$

$$\Lambda_{16} = -\frac{p_2}{2M_1} \left\{ \frac{p_2 l(y_0 - 1)}{2}(\theta_0 - \eta_0) + \frac{E_\nu^0}{E}Z_3 - \beta p_2 l(2ly_0 + p_2)\theta_3 - \beta X_2 - \frac{(X_3 - X_4)}{l} \right. \\ \left. + 2l\beta\zeta_{11} + \frac{\zeta_{21}}{2} + \frac{\beta}{2}\chi_{21} - \frac{\chi_{30}}{2l} + p_2^2(\gamma_0 - \eta_0) \right\},$$

$$\Lambda_{17} = \frac{p_2 l^2}{2M_1} \left[ \frac{\theta_7}{2} - E(\theta_3 - \theta_4) \right],$$

$$\Lambda_{18} = -\frac{p_2}{2M_1} \left\{ \frac{p_2 l(1 - y_0)}{2}(\eta_0 + \theta_0) - E[(1 - \beta^2)\zeta_{12} - \zeta_{11}] - 2EZ_2 \right. \\ \left. + l[(1 - \beta^2)\chi_{12} - \chi_{11}] + p_2^2(\gamma_0 - \eta_0 - \beta l\theta_3) - \frac{E + E_\nu^0}{E}Z_3 \right\},$$

$$\Lambda_{19} = -\frac{Ep_2}{2} \left\{ E_\nu^0 \left( -\frac{\gamma_0}{E} + \beta^2\theta_3 \right) - \frac{\eta_0}{E} \left( E - \frac{E_\nu^0}{2} \right) - (1 - \beta^2) \left[ \frac{\theta_6}{2} - E(\theta_2 - \theta_3) \right] \right. \\ \left. + (2 - \beta^2) \left[ \frac{\theta_7}{2} - E(\theta_3 - \theta_4) \right] + \frac{1}{4E} [\theta_9 - 2l^2\theta_3 + 6E^2(\theta_3 - \theta_4 - \beta\theta_5)] \right\},$$

$$\Lambda_{20} = \frac{2p_2 E^2}{M_1} \left\{ \eta_0 - (1 - \beta^2) \left[ \frac{\theta_7}{2} - E_\nu^0\theta_3 - E(\theta_3 - \theta_4) \right] - E_\nu^0(\theta_4 + \beta\theta_5) + \beta \left( \frac{\theta_{14}}{2} - l\theta_{10} \right) \right\},$$

$$\Lambda_{21} = \frac{p_2 E^2}{M_1} \left\{ -(1 - \beta^2) \left( \frac{\zeta_{11}}{E} + (E_\nu^0 + l\beta)\theta_3 \right) + (E_\nu^0 + \beta l)(\theta_4 + \beta\theta_5) + \frac{\zeta_{10}}{E} \right. \\ \left. + \beta^2 \left( -\eta_0 + p_2 Y_1 + \frac{\theta_{21}}{2} - l\theta_{20} \right) \right\},$$

$$\begin{aligned}
\Lambda_{22} &= \frac{Ep_2l}{M_1} \left\{ \frac{\zeta_{10}}{l} + \beta p_2 Y_1 + l Y_3 + \frac{\beta^2 - 1}{\beta} \left[ \frac{\zeta_{11}}{E} + \frac{\theta_7}{2} - E(\theta_3 - \theta_4) - \eta_0 \right] \right. \\
&\quad \left. + \left[ \frac{\theta_{14}}{2} - l\theta_{10} + \frac{\beta}{2}(\theta_{21} - 2l\theta_{20}) \right] \right\}, \\
\Lambda_{23} &= \frac{p_2 E^2}{M_1} \left\{ \beta^2 [E\theta_4 - (E + E_\nu^0)\theta_3 + \theta_0] + (1 + 2\beta^2)\eta_0 - (1 - \beta^2) \left[ \frac{\theta_7}{2} - E(\theta_3 - \theta_4) \right] \right. \\
&\quad \left. + \frac{\zeta_{10} - \zeta_{11}}{E} + \beta \left( \frac{\theta_{14}}{2} - l\theta_{10} \right) \right\}, \\
\Lambda_{24} &= \frac{p_2 E^2}{M_1} \left\{ \beta^2 [(E + E_\nu^0)\theta_3 - E\theta_4 - \theta_0] + \frac{\zeta_{11} - \zeta_{10}}{E} - (2\beta^2 + 1)\eta_0 \right. \\
&\quad \left. + (1 - \beta^2) \left[ \frac{\theta_7}{2} - E(\theta_3 - \theta_4) \right] - \frac{\beta}{2}\theta_{14} + \beta^2 E\theta_{10} \right\}, \\
\Lambda_{25} &= \frac{p_2 l^2}{M_1} \left[ E_\nu^0 \theta_4 + \theta_0 - (E_\nu^0 + \beta l)\theta_3 - \frac{\zeta_{11}}{E} \right], \\
\Lambda_{26} &= \frac{Ep_2}{4M_1} \left\{ -\frac{\zeta_{21}}{E} - 2(\zeta_{11} - \zeta_{10}) - (E_\nu^0 + \beta l)[\theta_7 - 2E(\theta_3 - \theta_4) - 2\eta_0] \right. \\
&\quad \left. + 2p_2 l y_0 (\theta_3 - \theta_4) - \beta \eta_0 \left[ l - \frac{p_2}{2}(y_0 - 1) \right] \right\}, \\
\Lambda_{27} &= \frac{p_2}{4M_1} \left\{ 2\beta l (p_2 l y_0 \theta_3 - \zeta_{11}) - E(2E_\nu^0 \beta^2 - E_\nu^0 + l\beta)[\theta_7 - 2E(\theta_3 - \theta_4)] + \zeta_{20} - \zeta_{21} \right. \\
&\quad \left. + l^2 \eta_0 - \frac{p_2 l (y_0^2 - 1)}{2} + E(\beta^2 - 1)[\theta_9 - 2l^2 \theta_3 + 6E^2(\theta_3 - \theta_4 - \beta \theta_5)] \right\}, \\
\Lambda_{28} &= \frac{p_2 l}{4M_1} \left[ E_\nu^0 (\theta_{14} - 2l\theta_{10}) + l\eta_0 + \frac{p_2 \eta_0}{2}(y_0 - 1) \right], \\
\Lambda_{29} &= -\frac{p_2 l \eta_0}{8M_1} [2l + p_2(y_0 - 1)].
\end{aligned}$$

As was mentioned earlier we are using the same notation as in Ref. [4]. There should arise no confusion, the list of  $\Lambda_i$  in this reference applies only to the  $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$  correlation and the above list applies only to the  $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$  correlation. However, the  $\theta_i$ ,  $\gamma_0$ ,  $\eta_0$ ,  $\zeta_{pq}$ ,  $\chi_{mn}$ , and the functions  $X_i$ ,  $Y_i$ , and  $Z_i$  given in Appendix B of that reference are common to both sets of  $\Lambda_i$ .

The  $\rho_i$  functions, after performing the integrations, become

$$\rho_I = \frac{2p_2 l}{M_1} (\chi_{11} - \chi_{10} - \beta \zeta_{11}),$$

$$\begin{aligned}
\rho'_I &= \frac{2p_2l}{M_1} \left[ \frac{p_2(y_0 - 1)}{2\beta^2} [2\beta^2\eta_0 + (\beta^2 - 1)\theta_0] + \frac{\zeta_{10} - \zeta_{11}}{\beta} \right. \\
&\quad \left. + \frac{\chi_{11} - \chi_{10}}{\beta^2} + EE_\nu^0\theta_5 + El\theta_{10} \right], \\
\rho_{II} &= \frac{p_2}{2M_1} \left[ \frac{E_\nu^0}{\beta} (\beta\zeta_{11} - \chi_{11} + \chi_{10}) - \frac{\zeta_{21}}{2} + \frac{\chi_{21} - \chi_{20}}{2\beta} \right], \\
\rho'_{II} &= -\frac{p_2}{2M_1} \left[ \frac{p_2l(1 - y_0)}{2} (\theta_0 + \eta_0) - p_2l^2(Y_1 - Y_5) - (E + E_\nu^0)(\zeta_{10} - \zeta_{11}) \right. \\
&\quad \left. + \beta p_2^2 l\mathcal{I} - l^2(\theta_0 - \beta p_2\theta_{12}) + \frac{\chi_{20} - \chi_{21}}{2\beta} + \frac{\zeta_{21}}{2} \right], \\
\rho_{III} &= \frac{p_2l}{M_1} \left[ \frac{m^2}{l} \left( \eta_0 - \frac{\theta_7}{2} + (E + E_\nu^0)(\theta_3 - \theta_4) \right) + \frac{1}{2}(E - 2E_\nu^0)(\theta_{14} - 2l\theta_{10}) \right. \\
&\quad \left. - EE_\nu^0\theta_5 - \frac{1}{2}p_2(y_0^2 - 1) + \beta p_2l\theta_{12} - l(\theta_0 + E_\nu^0\theta_4 - EY_3) \right], \\
\rho'_{III} &= \frac{p_2l}{M_1} \left[ \beta\zeta_{11} + p_2lY_1 + ElY_3 - \frac{m^2}{2l}\theta_7 - EE_\nu^0\theta_5 + \frac{E}{2}(\theta_{14} - 2l\theta_{10}) - l\theta_0 \right. \\
&\quad \left. + \frac{l}{2}(\theta_{21} - 2l\theta_{20}) + \frac{E}{\beta}(E + E_\nu^0)(\theta_3 - \theta_4) + \eta_0 \left( \frac{E}{\beta} - 2l + \frac{p_2}{2}(y_0 - 1) \right) \right].
\end{aligned}$$

As in the case of  $\Lambda_i$ , the use of the same notation for the  $\rho_i$  functions here and in Ref. [4] should not lead to confusions. Again both these sets use the same algebraic expressions on their right-hand sides. They can all be found in Appendix B of Ref. [4]. All the detailed definitions are found there.

Our results for  $\hat{s}_1 \cdot \hat{1}$  are general within our approximations. We covered the CDB and NDB cases. They can be used in the other four charge assignments of baryons involving heavy quarks and whether the charged lepton is  $e^\pm$ ,  $\mu^\pm$ ,  $\tau^\pm$ . They are model independent and are not compromised to fixing the form factors at prescribed values. They are given in two forms. The first one is the triple numerical integration form, in which the integrations over the real photon variables are explicitly exhibited and may be performed numerically. In the second one all those integrations were calculated analytically. Our calculations rely heavily on previous results and we have provided the references to the many expressions calculated before. Let us point out that there is a third way of using our RC results in the form of numerical arrays, as discussed in Ref.[4], which may be the more practical in experimental analyses. We shall not discuss this further, due to lack of space. The results will be useful in high statistics experiments of hyperon semileptonic

decays (involving only the light quarks  $u, d, s$ ) and of medium statistics experiments of heavy quark baryons (involving the transitions  $c \rightarrow s, d$  and  $b \rightarrow c, u, d$ ).

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